2.3-2.5 Derivative Rules: Quiz A Key

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General grading rules

- **Show work.** A bare correct answer earns at most half credit.
- **Any valid method.** Expanding first, negative exponents, the quotient rule, simplifying first, solving for y first where possible, or carrying the rules all the way through are all accepted; no method is required.
- **Notation.** y', $\dfrac{dy}{dx}$, $f'(x)$ and $\dfrac{d}{dx}[\ ]$ are all accepted.
- **Value vs. derivative.** A function value where a derivative was asked (or a first derivative where a second was asked) earns 0 for that part.
- **Simplifying.** Answers need not be simplified; equivalent forms are correct.
- **Excluded values.** Where a formula fails at some values, the key names them. No points are deducted for omitting statements of excluded values.
- **Trap notes** print the line that a skipped step leaves out, next to the answer the skip produces.

Quiz A

Item	Answer	Work, rubric, trap
1(a)	$2-4\pi^2$	$y' = 2x\cos 2x - 2x^2\sin 2x$; $y'' = 2\cos 2x - 4x\sin 2x - 4x\sin 2x - 4x^2\cos 2x = 2\cos 2x - 8x\sin 2x - 4x^2\cos 2x$; at π: $2-0-4\pi^2$. *Rubric:* 1 pt y'; 2 pts y''; 1 pt value. *Trap:* Writing $y' = 2x\cos 2x + 2x^2\sin 2x$ (the minus of $\dfrac{d}{dx}[\cos(2x)] = -2\sin(2x)$ kept in the head) gives $2+4\pi^2$. The line to write: $x^2\cdot(-\sin(2x))\cdot 2$. Dropping the inner 2 gives $2-2\pi^2$. Stopping at $y'(\pi) = 2\pi$ answers a different question (0 pts).
1(b)	0	$f' = -6(1-2x)^2$, $f'' = 24(1-2x)$, $f''' = -48$, $f^{(4)} = 0$ (or: a cubic, so the fourth derivative is 0). *Rubric:* 3 pts, all or nothing for 0 with work. *Trap:* $f^{(4)}$ is the fourth *derivative*; $(1-2x)^{12}$ or $4(1-2x)^3(-2)$ treats the (4) as a power. -48 stops one derivative early.
1(c)	-24	$y = 3(5-2x)^{-4}$; $y' = -12(5-2x)^{-5}\cdot(-2) = 24(5-2x)^{-5}$; at $x=3$: $5-6=-1$, $24(-1)^{-5} = -24$. *Rubric:* 1 pt outer derivative $-12(5-2x)^{-5}$; 1 pt inner -2; 1 pt value. *Trap:* Three minus signs: the exponent, the inner slope, and $(-1)^5$. Losing any one gives $+24$. The line usually skipped: $24(-1)^{-5} = 24\cdot(-1) = -24$.
1(d)	$8\sin(4x)\cos(4x)$ $+8x\cos(4x^2)$	$\sin^2(4x) = (\sin(4x))^2$: $2\sin(4x)\cdot\cos(4x)\cdot 4$. $\sin(4x^2)$: $\cos(4x^2)\cdot 8x$. Equivalent: $4\sin(8x) + 8x\cos(4x^2)$. *Rubric:* 1 pt first term; 1 pt second term; 1 pt both inner derivatives (4 and $8x$). *Trap:* $\sin^2(4x)$ (a square of a sine) and $\sin(4x^2)$ (a sine of a square) look alike; reading $\sin^2(4x)$ as $\sin(2\cdot 4x)$ gives $8\cos(8x)$.

Item	Answer	Work, rubric, trap
1(e)	0	Inside: $(t^2+4t+4)-(t^2-4t+4)-8t+9=9$, so $w(t)=3$ for every t and $w'(t)=0$. The chain rule also gives $\frac{1}{2}(\dots)^{-1/2}\,(2(t+2)-2(t-2)-8)=0$. *Rubric:* 3 pts, all or nothing for 0; either route. *Trap:* Distributing the minus to only the first term, $-(t-2)^2=-t^2-4t+4$, leaves $17-8t$ inside and a non-zero answer. Line to write: $-(t^2-4t+4)=-t^2+4t-4$.
2	-15	$h'=f'g+fg'$; $h''=f''g+f'g'+f'g'+fg''=f''g+2f'g'+fg''$. Row $x=2$: $4(-2)+2(-1)(5)+3(1)=-8-10+3=-15$. *Rubric:* 2 pts h'' formula (by differentiating h'); 1 pt correct table reads; 2 pts value. *Trap:* $f''(2)g''(2)=4$ is the product of second derivatives. Dropping the 2 gives -10. Columns are shuffled: x, f'', g, f', g'', f, g'.
3	-2	$g(-3)=2$, $g'(-3)=-1$ (dashed segment from $(-4,3)$ to $(-2,1)$); $f'(2)=2$ (solid segment from $(1,-3)$ to $(4,3)$). $k'(-3)=f'(g(-3))\,g'(-3)=(2)(-1)$. *Rubric:* 1 pt $g(-3)$ and $g'(-3)$; 2 pts f' read at 2, not at -3; 1 pt value. *Trap:* $f'(-3)\,g'(-3)=(-2)(-1)=2$ reads f' at -3; the sign flips. $f(g(-3))=f(2)=-1$ is a value, not a derivative.
4(a)	$-\frac{1}{5}$	$6x-2y-2xy'+6yy'=0$; $y'(6y-2x)=2y-6x$; $y'=\dfrac{2y-6x}{6y-2x}$; at $(1,2)$: $\dfrac{4-6}{12-2}=-\dfrac{1}{5}$. *Rubric:* 1 pt product rule on $-2xy$; 1 pt $6yy'$ and 0 on the right; 1 pt solving; 1 pt value. *Trap:* Numerator and denominator are mirror images; swapping them or copying a coefficient from the other factor gives -5. Writing $-2xy'$ as $+2xy'$ when moving it across "=" changes the sign. The 11 differentiates to 0, not 11.
4(b)	$-\dfrac{3y^2}{\sin x}$	Differentiate with respect to y: $-\sin(x)\,\dfrac{dx}{dy}=3y^2$, so $\dfrac{dx}{dy}=-\dfrac{3y^2}{\sin x}$. Excluded: $\sin x=0$. *Rubric:* 1 pt $-\sin(x)\dfrac{dx}{dy}$; 1 pt answer. *Trap:* $\dfrac{dy}{dx}=-\dfrac{\sin x}{3y^2}$ answers the drilled question; it is the reciprocal (0 pts). Losing the minus of $\dfrac{d}{dx}[\cos x]$ gives $+\dfrac{3y^2}{\sin x}$.
4(c)	-42	$3x^2+2yy'=0$, $y'=-\dfrac{3x^2}{2y}$ $(=-6$ at $(2,1))$. $y''=-\dfrac{6x\cdot 2y-3x^2\cdot 2y'}{(2y)^2}$; at $(2,1)$: $-\dfrac{24-12\cdot 2\cdot(-6)}{4}=-\dfrac{24+144}{4}=-42$. *Rubric:* 1 pt y'; 2 pts quotient rule with y' inside; 1 pt value. *Trap:* Putting $(2,1)$ into y' first gives the number -6, whose derivative is 0; substitute only after differentiating. With $y'=-6$, the line to write is $-3x^2\cdot 2y'=-12\cdot 2\cdot(-6)=+144$; losing that sign gives $-\dfrac{24-144}{4}=30$.
5	$\frac{2}{3}$	Product rule: $g(y)+x\,g'(y)\,y'=0$. The point has $y=2$, so use row 2: $g(2)=4$, $g'(2)=-2$. $4+3(-2)y'=0$, $y'=\frac{4}{6}=\frac{2}{3}$. *Rubric:* 1 pt product rule; 1 pt $g'(y)\,y'$; 1 pt reads at $y=2$; 2 pts value. *Trap:* The table's column says x, and 3 is the point's x, but g is applied to y. Row 3 throughout gives $\frac{1}{12}$; $g(2)$ with $g'(3)$ gives $-\frac{1}{3}$. Check: $3\,g(2)=12$ confirms the row.

Points: 40.